GRADE 8 MATHEMATICS

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to: (B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

8.5B INSTRUCTIONAL LESSON & ASSESSMENT

For this TEKS students should be able to find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

BEFORE THE LESSON:

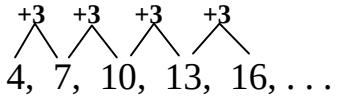
1. Make teacher transparencies.
2. Make copies of Student Activity sheets, Open-Ended and Mini-Assessment.
3. **Materials for Student Activity #3:** 1 set of 50 color tiles per pair of students, 1 sheet of centimeter grid paper per pair of students, 1 graphing calculator per student, Student Activity #3

MATH BACKGROUND

Understanding how to use an algebraic expression to represent any term in an arithmetic sequence
A **sequence** is a set of numbers written in a particular order. For example, 1, 9, 25, 49 is a **sequence** of four numbers. The number 1 is the first term in the sequence, 9 is the second term, 25 is the third term, and 49 is the fourth term.

An **arithmetic sequence** is a sequence of *n* numbers where the difference between the successive terms is constant. For example: The first five terms of an **arithmetic sequence** are 4, 7, 10, 13, 16 …. The number 4 is the first term in the sequence, 7 is the second term, 10 is the third term, and 13 is the fourth term.

The common difference in an **arithmetic sequence** can be identified by finding the difference between the terms in the sequence.



In the sequence 4, 7, 10, 13, 16 . . ., the common difference is 3. The value of the term in position 1 is 4. Multiply the common difference times the term's position.

common difference

$$3 \cdot 1 = 3$$

The term in position 1 is 4, not 3, therefore you must add or subtract from 3 to find the value of the term in position 1.

$$3 + 1 = 4$$

The term in position 2 is 7 and $(3 \cdot 2) + 1 = 7$. The term in position 3 is 10 and $(3 \cdot 3) + 1 = 10$.

A rule can be used to find the *n*th term in this arithmetic sequence. Multiply 3 times the position of the term and add 1.

The rule can be expressed algebraically as $3n + 1$.

A rule can be used to find the *n*th term in any arithmetic sequence. The *n*th term in an arithmetic sequence can be found by multiplying the common difference by *n*, the position of the term in the sequence, and adding or subtracting from the product to get the correct value of the term. The rule can be expressed algebraically.

Follow these guidelines to find a rule or expression that can be used to find the *n*th term in an arithmetic sequence:

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.
- State the pattern as a rule.
- Check to see whether the rule works for the next two terms in the sequence.
- Represent the rule as an algebraic expression.

What expression can be used to find the *n*th term in this sequence?

4, 8, 12, 16, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position	1	2	3	4	. . .	*n*
Value of Term	4	8	12	16	. . .	?

+4 +4 +4

- State the pattern as a rule.

  The 1st term is $4 \cdot 1 = 4$. Maybe each term in this sequence is equal to 4 times its position number in the sequence.

- Check to see whether the rule works for the next two terms in the sequence.

  The 2nd term is $4 \cdot 2 = 8$.

  The 3rd term is $4 \cdot 3 = 12$.

- Represent the rule as an algebraic expression.

  The value of the *n*th term is $4 \cdot n$, or $4n$.

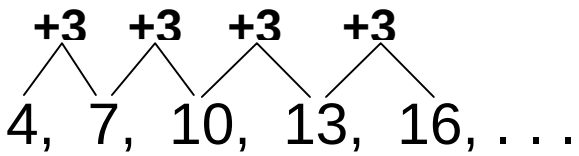
8.5B INSTRUCTIONAL ACTIVITY #1

A **sequence** is a set of numbers written in a particular order. For example, 1, 9, 25, 49 is a sequence of four numbers. The number 1 is the first term in the sequence, 9 is the second term, 25 is the third term, and 49 is the fourth term.

An **arithmetic sequence** is a sequence of *n* numbers where the difference between the successive terms is constant.

For example: The first five terms of an **arithmetic sequence** are 4, 7, 10, 13, 16 …. The number 4 is the first term in the sequence, 7 is the second term, 10 is the third term, and 13 is the fourth term.

The common difference in an **arithmetic sequence** can be identified by finding the difference between the terms in the sequence.



In the sequence 4, 7, 10, 13, 16 . . ., the common difference is 3. The value of the term in position 1 is 4. Multiply the common difference times the term's position.

common difference

$3 \bullet 1 = 3$

The term in position 1 is 4, not 3, therefore you must add or subtract from 3 to find the value of the term in position 1.

$3 + 1 = 4$

The term in position 2 is 7 and $(3 \bullet 2) + 1 = 7$.
The term in position 3 is 10 and $(3 \bullet 3) + 1 = 10$.

A rule can be used to find the *n*th term in this arithmetic sequence. Multiply 3 times the position of the term and add 1.

The rule can be expressed algebraically as $3n + 1$.

A rule can be used to find the *n*th term in any arithmetic sequence. The *n*th term in an arithmetic sequence can be found by multiplying the common difference times *n*, the position of the term in the sequence, and adding or subtracting from the product to get the correct value of the term. The rule can be expressed algebraically.

Follow these guidelines to find a rule that can be used to find the *n*th term in an arithmetic sequence:

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.
- State the pattern as a rule. Example: each term in this sequence is equal to 3 times its position.
- Check to see whether the rule works for the next two terms in the sequence.
- Represent the rule as an algebraic expression. Example: 3*n.*

What rule can be used to find the *n*th term in this sequence?

4, 8, 12, 16, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position	1	2	3	4	. . .	*n*
Value of Term	4	8	12	16	. . .	?

+4 +4 +4

- State the pattern as a rule.
  The common difference is 4.

  The 1st term is 4 • 1 = 4. Maybe each term in this sequence is equal to 4 times its position number in the sequence.

- Check to see whether the rule works for the next two terms in the sequence.

  The 2nd term is 4 • 2 = 8.

  The 3rd term is 4 • 3 = 12.

- Represent the rule as an algebraic expression.
  The value of the *n*th term is 4 • *n*, or 4*n*.

8.5B STUDENT ACTIVITY #1

Problem #1: What expression can be used to find the *n*th term in the following arithmetic sequence where *n* represents a number's position in the sequence?

5, 10, 15, 20, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position					. . .	*n*
Value of Term					. . .	?

- State the pattern as a rule.

  The common difference is ____. Use the _____ _____ to find the rule.

  Using the common difference, the 1st term is ____ • 1 = 5. The term in position 1 is ____,

  Maybe each term in this sequence is equal to ____ times its _____ number in the _____.

- Check to see whether the rule works for the next two terms in the sequence.

  The 2nd term is ____ • 2 = ____. The rule works for the _____ term.

  The 3rd term is ____ • 3 = ____. The rule works for the _____ term.

- Represent the rule as an algebraic expression.

  The value of the *n*th term is ____ • *n*, or ____*n*.

Problem #2: What is the 15th term in this sequence?

2, 6, 10, 14, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position	**Value of Term**
n	?

- State the pattern as a rule.

  The common difference is ____. Use the _____ _____ to find the rule.

  Using the common difference, the 1st term is ____ • 1 = 4. The term in position 1 is ____, not 4, therefore you must add or subtract from _____ to find the value of the term in position 1.

Maybe each term in this sequence is equal to ____ times its _____ number in the _____, subtract 2.

___(1) – ___ = ___ – ___ = ___ Does the rule work for the first term? _____

- Check to see whether the rule works for the next two terms in the sequence.

___(2) – ___ = ___ – ___ = ___ Does the rule work for the second term? _____

___(3) – ___ = ___ – ___ = ___ Does the rule work for the third term? _____

- Represent the rule as an algebraic expression.

The *n*th term in the sequence is ___*n* – ___.

- Find the value of the 15th term in the sequence.

For the 15th term in the sequence, *n* is ____.

Substitute ____ for *n* in the rule ___*n* – ___.

____(15) – ____ = ____ – ____ = ____

The 15th term in the sequence is _____.

Problem #3: What expression can be used to find the *n*th term in the following arithmetic sequence where *n* represents a number's position in the sequence?

1.5, 3, 4.5, 6, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position					. . .	*n*
Value of Term					. . .	?

- State the pattern as a rule.

The common difference is ____. Use the _____ _____ to find the rule.

The 1st term is ____ • 1 = ____. Maybe each term in this sequence is equal to ____ times its _____ number in the _____.

- Check to see whether the rule works for the next two terms in the sequence.

The 2nd term is ____ • 2 = ____.

The 3rd term is ____ • 3 = ____.

- Represent the rule as an algebraic expression.

The value of the *n*th term is ____ • *n*, or ____*n*.

8.5B INSTRUCTIONAL ACTIVITY #2

If the terms of the arithmetic sequence decrease in value the common difference is a negative number.

In the sequence 20, 16, 12, 8, 4, . . ., the common difference of –4.

What is the 15th term in this sequence?

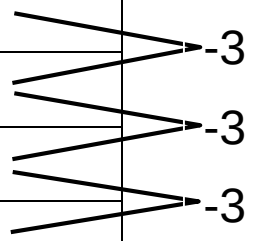
14, 11, 8, 5, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position	Value of Term
1	14
2	11
3	8
4	5
n	?

-3
-3
-3

Position	Value of Term
1	14
2	11
3	8
4	5
n	?



- State the pattern as a rule.

  The common difference is –3.

  The 1st term is 14. Maybe each term in this sequence is equal to –3 times its position number in the sequence.

  If n is 1, then $-3 \cdot n = -3$ and the first term is 14, not –3.

  The pattern does not work.

  Try a different rule. Add or subtract from the product of the common difference and the position to get the correct value of the term.

Maybe each term in this sequence is equal to –3 times its position number in the sequence, add 17.

The rule works for the 1st term.
$-3(1) + 17 = -3 + 17 = 14$

- Check to see whether the rule works for the next two terms in the sequence.

The rule works for the 2nd term.
$-3(2) + 17 = -6 + 17 = 11$

The rule works for the 3rd term.
$-3(3) + 17 = -9 + 17 = 8$

- Represent the rule as an algebraic expression.

The nth term in the sequence is $-3n + 17$.

- Find the value of the 15th term in the sequence.

For the 15th term in the sequence, n is 15.
Substitute 15 for n in the rule $-3n + 17$.

$$-3(15) + 17 = -45 + 17 = -28$$

The 15th term in the sequence is –28.

8.5B STUDENT ACTIVITY #2

Problem #1: What expression can be used to find the *n*th term in this sequence?

1.25, 1, 0.75, 0.5, ...

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position	Value of Term
n	?

- State the pattern as a rule.

  The common difference is _____. Use the _____ _____ to find the rule.

  If n is 1, then $-0.25 \cdot n = -0.25$ and the first term is 1.25, not -0.25. The pattern does ___ work.

  Try a different rule. _____ or _____ from the product of the common difference and the position to get the correct value of the term.

  Maybe each term in this sequence is equal to -0.25 times its position number in the sequence, add _____.

  $-0.25(1)$ + _____ = -0.25 + _____ = _____ Does the rule work for the first term? ____

- Check to see whether the rule works for the next two terms in the sequence.

  $-0.25(2)$ + _____ = _____+ _____ = _____ Does the rule work for the second term? ____

  $-0.25(3)$ + _____ = _____+ _____ = _____ Does the rule work for the third term? ____

- Represent the rule as an algebraic expression.

  The *n*th term in the sequence is $-0.25n$+ _____.

- Find the value of the 14th term in the sequence.

  For the 14th term in the sequence, n is 14. Substitute 14 for n in the rule $-0.25n$+ _____.

  $-0.25\ (14)$ + _____ = _____ + _____ = _____

  The 14th term in the sequence is _____.

Problem #2: Look at this sequence of numbers.

$$-4,\ -6,\ -8,\ -10.\ -12$$

Does the expression $-2n-2$ show the relationship between any term and *n*, its position in the sequence?

Complete the table to find if this rule works for all the terms in the sequence.

Position	$-2n-2$	**Value of Term**	**Correct?**
1	$-2(1)-2 = -4$	-4	Yes
2			
3			
4			
5			

For the sequence $-6,\ -8,\ -10,\ -12\ -14$, . . ., the expression _____ shows the relationship between any term and ____, its position in the sequence.

Problem #3: Let n represent the position of a number in the following arithmetic sequence. What expression can be used to find any term in the sequence?

$$\frac{1}{2},\ 1,\ \frac{3}{2},\ 2,\ ...$$

- Use the common difference to find a pattern that shows the relationship between a term's position number and the value of the term.

Position					. . .	*n*
Value of Term					. . .	?

- State the pattern as a rule.

  The common difference is _____. Use the _____ _____ to find the rule.

  The 1st term is ____ • 1 = ____. Maybe each term in this sequence is equal to $\frac{\square}{\square}$ times its _____ number in the _____.

- Check to see whether the rule works for the next two terms in the sequence.

  The 2nd term is $\frac{\square}{\square} \bullet 2 = \frac{\square}{\square} =$ _____.

  The 3rd term is $\frac{\square}{\square} \bullet 3 = \frac{\square}{\square}$.

- Represent the rule as an algebraic expression.

  The value of the *n*th term is $\frac{\square}{\square} \bullet n$ *or* $\frac{\square}{\square} n$, or $\frac{n}{\square}$.

Problem #4: Which of the following algebraic expressions best describes the *n*th term in the sequence $2\frac{1}{3}$, $2\frac{2}{3}$, 3, $3\frac{1}{3}$, ..., where *n* represents a term's position in the sequence?

$\frac{n}{3} + 2$ $\qquad$ $2n + \frac{1}{3}$ $\qquad$ $\frac{7n}{3}$

Check each expression for the values of *n* given in the table. Identify the expression(s) that produces the correct term in each case.

Position	**Rule #1** $\frac{n}{3} + 2$	**Rule #2** $2n + \frac{1}{3}$	**Rule #3** $\frac{7n}{3}$	**Value of Term**	**Which Rule(s) Give the Correct Value?**
1	$\frac{1}{3} + 2 = 2\frac{1}{3}$	$2(1) + \frac{1}{3} =$ $2 + \frac{1}{3} = 2\frac{1}{3}$	$\frac{7(1)}{3} =$ $\frac{7}{3} = 2\frac{1}{3}$	$2\frac{1}{3}$	Rule #1 Rule #2 Rule #3
2				$2\frac{2}{3}$	
3				3	
4				$3\frac{1}{3}$	

The expression _____ describes the *n*th term in the sequence because it works for all four numbers given.

The expressions _____ and _____do not describe the *n*th term in the sequence because they do not work for all four numbers given.

8.5B INSTRUCTIONAL ACTIVITY #3

MATERIALS: 1 set of 50 color tiles per pair of students, 1 sheet of centimeter grid paper per pair of students, 1 graphing calculator per student, Student Activity #3

PROCEDURE:

- Students complete Student Activity #3 in partner pairs.
- Read the following scenario to students while displaying the transparency on page 17.

  A floor tile designer created these patterns: Can you write a rule that will help the designer find the number of tiles needed for the nth figure in order to continue this pattern?

Distribute the materials for Student Activity #3 and before students begin working, ask the following questions:

- How will you determine how many tiles will be needed for the fifth figure?

During Student Activity #3, roam the room and listen for the following:

- Do the students discuss how the pattern continues?
- Can the students verbalize the rule?
- Do the students check their work for reasonableness without a graphing calculator?
- Do the students check their work for reasonableness with a graphing calculator?
- Can the students make correct predictions based on the rule?
- Do the students use appropriate vocabulary?
- Do the students justify the rule?
- Can the students differentiate the term number from the number of tiles?

During Student Activity #3, roam the room and look for the following:

- Do the students organize information?
- Can the students use a systematic process to develop the rule?
- Do the students draw additional figures?
- Do the students use symbols to describe the pattern or rule?
- Do the students make a table?
- Do the students recognize flaws or errors in the rule and adjust it accordingly?
- Do the students use the calculator appropriately?

Answers to these questions can be used to support decisions related to further whole class instruction or group and individual student instruction during tutorial settings.

DESIGNER FLOOR TILE PATTERN

A floor tile designer created this pattern: Can you write a rule that will help the designer find the number of tiles needed for the nth figure in order to continue this pattern?

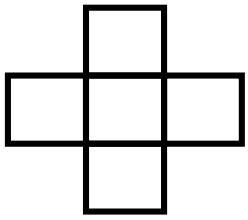


Figure 1

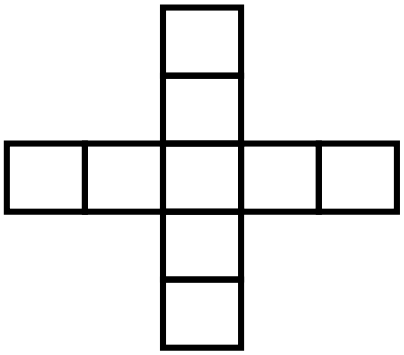


Figure 2

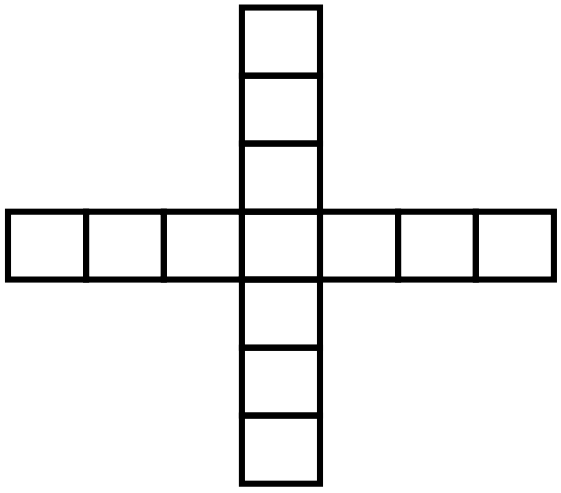


Figure 3

8.5B STUDENT ACTIVITY #3

DESIGNER TILE PATTERN

Problems: How will you determine how many tiles will be needed for the fifth figure?
How will you determine how many tiles will be needed for the sixth figure?
How will you determine how many tiles will be needed for the twelfth figure?
How will you determine how many tiles will be needed for the 100th figure?

Materials: 1 set of 100 color tiles per pair of students
1 sheet of centimeter grid paper per pair of students

Procedure: Work in partner pairs for this activity.

Scenario:
A floor tile designer created this pattern: Can you write a rule that will help the designer find the number of tiles needed for the nth figure in order to continue this pattern?

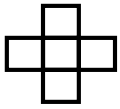


Figure 1

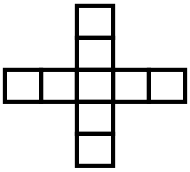


Figure 2

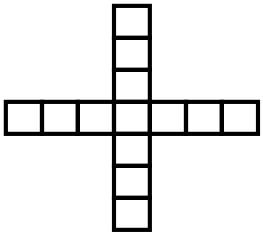


Figure 3

- Work with your partner to decide how to find how many tiles are needed for the fifth, sixth, twelfth, hundredth, and nth figure in the pattern. Your tools are color tiles, grid paper and graphing calculators.

Answer the following questions:

- What patterns do you see?

- Do you need to draw every figure from figure 4 to figure 100? Why or why not?

- What are some more efficient ways than drawing?

- Describe the pattern in words.

- Describe the pattern in symbols.

- Write a rule to describe the pattern.

- What is the relationship between the figure number and the number of tiles needed?

- Can you use this relationship to help you find the number of tiles needed for the fourth and fifth figures? Explain your answer.

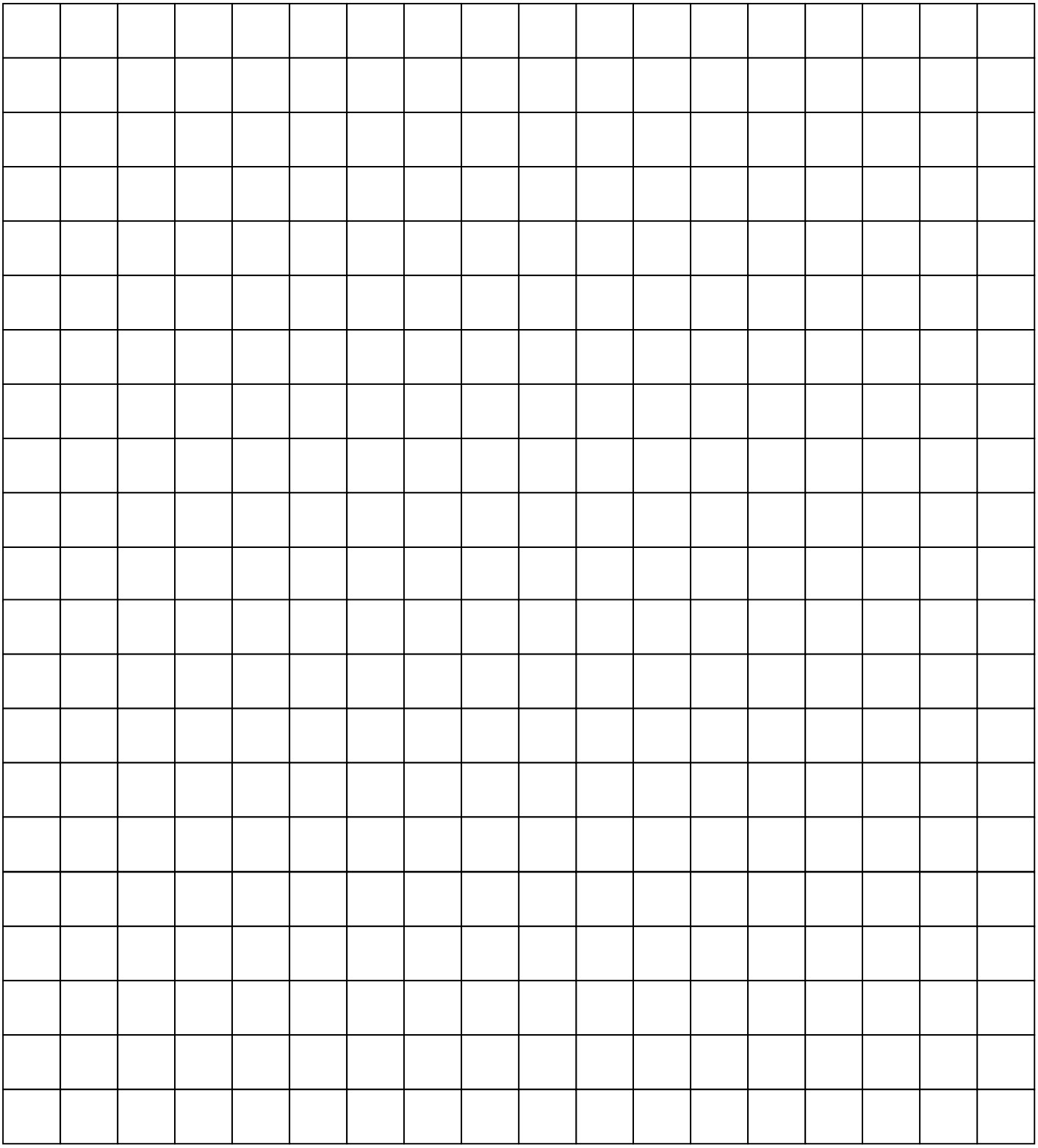
- What generalizations can you make?

- If the figure has 29 tiles what figure number is it? Explain your answer.

- If the figure has 229 tiles what figure number is it? Explain your answer.

- Use a graphing calculator to test your rule. Explain how you used your graphing calculator.

Centimeter Grid



8.5B OPEN-ENDED #1

The 3rd term of an arithmetic sequence is 15 and the 5th term is 23.

- Find the value of the first term. Show your work.

- Write an expression to represent the *n*th term of the sequence.

.

1. What mathematical concepts and vocabulary do I need to know to be able to work this problem?

2. Will the Grade 8 Mathematics Formula Chart be helpful on this problem? Why or why not?

3. Will a table of values help me with this problem? Why or why not?

4. What problem-solving strategy or strategies will I use to help solve this problem?

5. **Extension** (8.4A)**:** Make a scatterplot for the first five terms of the sequence.

8.5B OPEN-ENDED #1

The 3rd term of an arithmetic sequence is 15 and the 5th term is 23.

- Find the value of the first term. Show your work.

- Write an expression to represent the *n*th term of the sequence.

GRADE 8 MATHEMATICS

(8.5) Patterns, relationships, and algebraic thinking. The student uses graphs, tables, and algebraic representations to make predictions and solve problems. The student is expected to: (B) find and evaluate an algebraic expression to determine any term in an arithmetic sequence (with a constant rate of change).

NAME_____ DATE_____ SCORE ___/5

8.5B Mini-Assessment

1. In the sequence below, which expression can be used to find the value of the term in the *n*th position?

Position	Value of Term
1	0.125
2	0.25
3	0.375
4	0.5
5	0.625
n	

A $n - 0.875$

B $8n$

C $\frac{1}{8}n$

D $n^2 \div 8$

2. If *n* is the position of a number in this sequence, which expression identifies this pattern?

Position in Sequence	1	2	3	4	5	*n*
Term	2.5	5	7.5	10	12.5	?

A $n + 1.5$

B $2.5n$

C $n \div 2.5$

D $2n + 0.5$

3. Let *n* represent the position of a number in an arithmetic sequence.

0.5, 2.5, 4.5, 6.5,...

Which expression can be used to find any term of the sequence?

A $\frac{4n-3}{2}$

B $\frac{4n+3}{2}$

C $\frac{n}{2}$

D $\frac{3n}{2} - 1$

4. The expression below describes a pattern of numbers.

$3n + 6$

If *n* represents the position of a number in the pattern, which of the following is the pattern of numbers described by the expression?

A {1, 9, 12, 15, 18, ...}

B {6, 12, 22, 36, 54, …}

C {9, 12, 15, 18, 21,…}

D {3, 9, 12, 15, 21,…}

5. Let *n* represent the position of a term in a sequence. An arithmetic sequence is represented by the expression $5n + 2$. Which of the following statements is a true statement?

A The 10th term of the sequence is 50.

B The 5th term of the sequence is 28.

C The 30th term of the sequence is 150.

D The 100th term of the sequence is 502.

NAME_____ DATE_____ SCORE ___/5

8.5B Homework #1

1. Let *n* represent the position of a number in a sequence. Write the first five terms in the arithmetic sequence represented by the expression $2n+4$.

   _____, _____, _____, _____, _____

2. Let *n* represent the position of a number in a sequence. Write the first five terms in the arithmetic sequence represented by the expression $\frac{2n+4}{5}$.

   _____, _____, _____, _____, _____

3. Describe in words the relationship between the value of the term and the term's position in this arithmetic sequence:

   5, 10, 15, 20, 25, ...

   _____

   _____

4. Let *n* represent the position of a number in a sequence. A sequence is represented by the expression, $5n+1$. Give the value of the terms in the following positions. Show your work.

   - The tenth term:
   - The 15th term:
   - The 30th term:

5. Let *n* represent the position of a number in a sequence. A sequence is defined by the expression $4n-3$. To determine if a term would belong in this sequence, you would add 3 to the term. If the sum is a multiple of 4, the term belongs in this sequence. Circle the values below that belong in this sequence. Show your work.

   21 37 52 76 101

NAME_____ DATE_____ SCORE ___/5

8.5B Homework #2

1. Fill in the missing terms of this arithmetic sequence. Show your work.

   8, _____, 14, 17, _____, _____

2. Let *n* represent the position of a term in a sequence. Write the first five terms of the sequence represented by the expression $\frac{n+4}{5}$. Show your work.

   _____, _____, _____, _____, _____

3. Describe in words the relationship between the value of the term and the term's position in this sequence:

   3, 5, 7, 9, 11, ...

   _____

   _____

4. Let *n* represent the position of a term in a sequence. A sequence is represented by the expression $0.5n+1.5$. Give the value of the terms in the following positions. Show your work.

   - The tenth term:
   - The 15th term
   - The 30th term

5. Let *n* represent the position of the terms in a sequence. A sequence is represented by the expression $7n+1$. To determine if a term would belong in this sequence, you would subtract 1 from the term. If the difference is a multiple of 7, the term belongs in this sequence. Circle the terms that belong in this sequence.

   36 57 65 78 98